

Chapter 7

Q5:

$$KE = \frac{1}{2} m v^2$$

$$\boxed{K_{Es} = \frac{1}{2} m v_s^2} \quad \text{--- (1)}$$

$$\boxed{K_{Ef} = \frac{1}{2} (2m) v_f^2 = m v_f^2} \quad \text{--- (2)}$$

$$K_{Ef} = \frac{1}{2} K_{Es}$$

$$m v_f^2 = \frac{1}{2} \left(\frac{1}{2} m v_s^2 \right)$$

$$4 v_f^2 = v_s^2 \Rightarrow \underline{v_s = 2 v_f}$$

$$K'_{Ef} = K'_{Es}$$

$$\frac{1}{2} (2m) (v_{f+1})^2 = \frac{1}{2} m v_s^2$$

$$(v_{f+1})^2 = \frac{v_s^2}{2}$$

$$(v_{f+1})^2 = \frac{4v_f^2}{2} \Rightarrow \sqrt{(v_{f+1})^2} = 2v_f$$

$$v_{f+1} = \sqrt{2} v_f$$

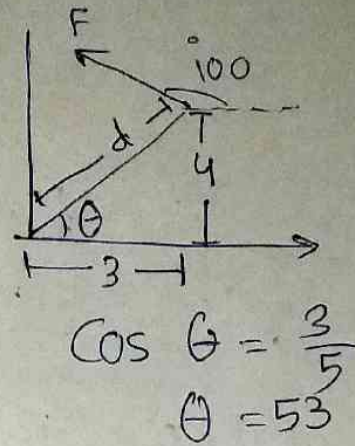
$$1 = \sqrt{2} v_f - v_f$$

$$\frac{1}{(\sqrt{2}-1)} = \frac{v_f (\sqrt{2}-1)}{(\sqrt{2}-1)}$$

a) $v_f = 2.4 \text{ m/s}$

b) $v_s = v_f * 2 = 4.8 \text{ m/s}$

Q10: $|F| = 2.5 \text{ N}$
 $|d| = 5 \text{ m}$
 $100 - \theta = \Phi$
 $100 - 53 = 47$
 $\omega = |5| |2.5| \cos 47$
 $= 8.5 \text{ J}$

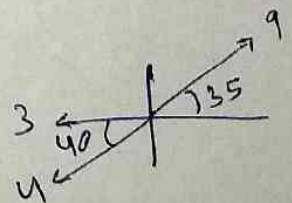


Q11: $d = (5\hat{i} - 3\hat{j} + 4\hat{k}) \text{ m}$ $|d| = \sqrt{5^2 + 3^2 + 4^2}$
 $|F| = 22 \text{ N}$
 $\theta = ?$
 $\omega = \Delta k$

(a) $7 * 22 \cos \theta = 45.0$
 $\cos \theta = 0.289$
 $\theta = 73.18^\circ$

(b) $7 * 22 \cos \theta = -45.0$
 $\theta = 106.9^\circ$

Q14:
 $F_x = 9 \cos 35 - 3 - 4 \cos 40 = 1.3$
 $F_y = 9 \sin 35 - 4 \sin 40 = 2.59$
 $F = \sqrt{(1.3)^2 + (2.6)^2} = 2.89 \text{ N}$
 $\omega = 2.9 * 4 = 11.6 \text{ J}$



Q16 :

$x = 5\text{ m}$	$k = 0$
$x = 0$	$k = 30\text{ J}$

$$k - k_0 = \left(\frac{\Delta k}{\Delta x}\right) * (x - x_0)$$

$$k - 30 = \left(\frac{30 - 0}{0 - 5}\right) (x - 0)$$

$$k(x) = -6x + 30$$

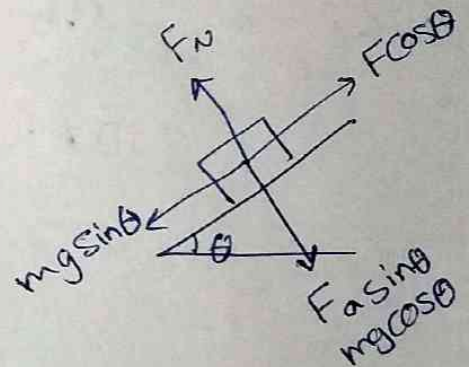
$$k_s = \frac{1}{2} m v^2 \Rightarrow v = \sqrt{\frac{2k_s}{m}} = \sqrt{\frac{2 * 48}{7}} = 3.7\text{ m/s}$$

Q24:

$$W = (F \cos \theta - mg \sin \theta) d$$

$$W = (23 \cos 30 - 10 * 3 \sin 30) 0.58$$

$$W = 2.85\text{ J}$$



$$W = \Delta k_s$$

$$k_{s1} = \text{zero}$$

$$W = k_{s2} - k_{s1} \Rightarrow 2.85 = \frac{1}{2} m v^2$$

$$2.85 = \frac{1}{2} * 3 * v^2$$

$$v = 1.3\text{ m/s}$$

... from the spring.

Q26:

$k = 5000$

$x_i = 0.05 \text{ m}$

$x_f = 0.1 \text{ m}$

$$W_{\text{required}} = W_{\text{spring}} = \frac{\Delta \frac{1}{2} k (\Delta x)^2}{2} = \frac{1}{2} (5000) (0.1^2 - 0.05^2)$$

$$= 18.75 \text{ J}$$

Q34:

$$w = \int F dx = m \int a dx = 15 \left(\frac{1}{2} \times 8 \times 24 \right) = 1440 \text{ J}$$

Q36:

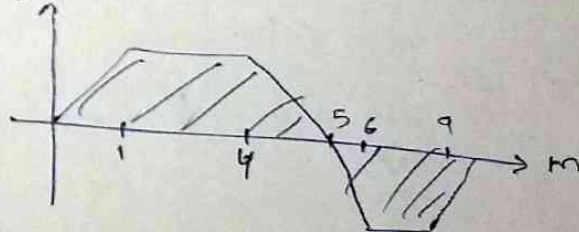
the area bound the curve = w

$$10 \times 2 + \frac{1}{2} \times 2 \times 0 - \frac{1}{2} \times 2 \times 5 = w$$

$$w = 30 - 5 = 25 \text{ J}$$

Q37:

$F = m \times a \text{ (N)}$



a) $w \Big|_{x=4} = \int_0^4 F x dx = \frac{1}{2} \times 1 \times 1 + 1 \times 12 + 1 + 3 \times 12 = 42 \text{ J}$

b) $w \Big|_{x=7} = 42 + \frac{1}{2} \times 12 \times 1 - \frac{1}{2} \times 12 \times 1 - 1 \times 12 = 30 \text{ J}$

c) $w \Big|_{x=9} = 30 - 1 \times 12 - \frac{1}{2} \times 12 \times 1 = 12 \text{ J}$

d) $w \neq \Delta K_s \rightarrow \frac{1}{2} m v^2 = w \rightarrow 42 = \frac{1}{2} \times 2 \times v^2 \rightarrow v = \sqrt{42} = 6.4 \text{ m/s}$

e) $v = \sqrt{30} \rightarrow 5.5 \text{ m/s}$

f) $v = \sqrt{12} \rightarrow 3.5 \text{ m/s}$

Q38:

a) $\Delta K_i = W$

$$K_{s2} - K_{s1} = \int_0^2 (2.5 - x^2) dx$$

$$K_{s2} = 2.5x - \frac{x^3}{3} \Big|_0^2$$

$$K_{s2} = 2.33 \text{ J}$$

$m = 1 \text{ kg}$
 $v_i = 0$
 $F = (2.5 - x^2) \text{ N}$

b) $W = \int F \cdot dx$

$W = F(x) \rightarrow F(x) = 0$

$$2.5 - x^2 = 0$$

$$2.5 = x^2$$

$$\rightarrow x = 1.58 \text{ m}$$

$$K_f - 0 = \int_0^{1.58} (2.5 - x^2) dx$$

$$K_f = (2.5x - \frac{x^3}{3}) \Big|_0^{1.58}$$

$$= 2.63 \text{ J}$$

Q39:

$W = \Delta K$

$$|V_1| = \sqrt{5^2 + 18^2} = 18.6 \text{ m/s}$$

$$|V_2| = \sqrt{9^2 + 22^2} = 23.7 \text{ m/s}$$

$$\Delta K = \Delta(\frac{1}{2} mV^2) = \frac{1}{2} \times 0.2 (23.7^2 - 18.6^2) = 2.2 \text{ J}$$

Q40:

$$W = \int F dx$$

$$= \int_{0.25}^{2.25} e^{-4x^2} dx = 0.212 \text{ J}$$

Q. 1

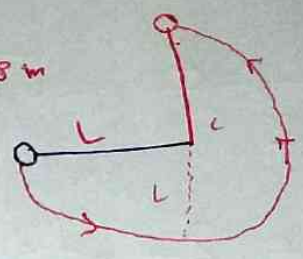
Adam stretches the spring x meter
John stretches the spring $3x$ meter

$$\frac{U_{s \text{ Adam}}}{U_{s \text{ John}}} = \frac{\frac{1}{2} k x^2}{\frac{1}{2} k (3x)^2} = \frac{x^2}{9x^2} = \frac{1}{9}$$

Q. 4
a)

$m = 382 \text{ kg}$

$L = 0.498 \text{ m}$

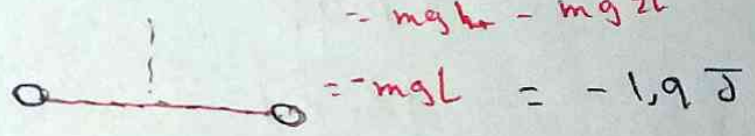


$W_g = -\Delta U$
 $W_g = (U_i - U_f) = mgh_i - mgh_z = 382 \times 10 \times 0.498 = 1.9 \text{ J}$

b) at the highest point

$W_g = -\Delta U$
 $= -(U_f - U_i) = U_i - U_f = mgh_i - mgh_z$
 $= mgh_i - mgh_z$

c) at the point in the right



$W_g = -\Delta U = U_i - U_f = mgh_i - mgh_z = mgL - mgL = \text{Zero}$

d) U_g at lowest point

$U_g = mgh = -1.9 \text{ J}$
 because at the initial point $U = \text{Zero}$

e) $U_g = mgh = mgL = 1.9 \text{ J}$

f) $U_g = mgh = \text{Zero}$

because it's at the same level with initial point



g) $E_{\text{mec}} = \Delta K + \Delta U_g$ / At highest point " $v=0$ "

then $\Delta K = 0$

$\Delta U_g = E_{\text{mec}} \Rightarrow$ then ΔU_g would be greater

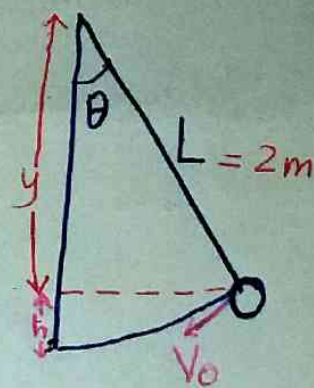
⑦ $m = 5 \text{ Kg}$, $L = 2 \text{ m}$, $V_0 = 0$, $\theta = 30^\circ$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{y}{L} = \frac{\sqrt{3}}{2}$$

$$y = \frac{\sqrt{3} \cdot 2}{2} = \sqrt{3} = 1.73 \text{ m}$$

$$h = L - y = 2 - 1.73 = 0.27 \text{ m}$$



⑧ $W = -\Delta U_g = -(U_f - U_i)$
 $= U_i - U_f$
 $= + U_f$
 $= + mgh$
 $= 13.39 \text{ J}$

⑨ $W = -\Delta U_g$
 $\Delta U_g = -W = -13.39 \text{ J}$

⑩ $\Delta E_{\text{mec}} = \Delta U + \Delta K$
 $0 = (U_f - U_i) + (K_f - K_i)$

$$K_f = U_i$$

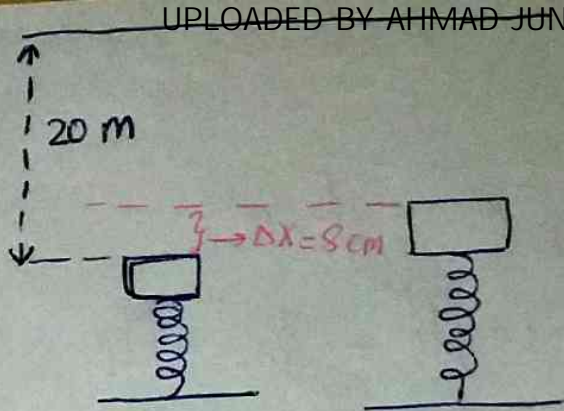
$$\frac{1}{2} mV^2 = 13.39$$

$$V = 2.3 \text{ m/s}$$

⑪ If θ increase \Rightarrow the height increase
 $\Rightarrow mgh$ increase
 \Rightarrow then ΔU_g increase

(لأن طاقة الوضع عند أسفل نقطة = صفر)
 وبالتالي تعتمد على طاقة الوضع الابتدائية "U_i"
 وبالتالي جميع القيم سوف تزداد

13) $m = 5g = 0.005 \text{ Kg}$



a) $\Delta U_g = U_f - U_i$
 $= mgh = 0.005 \times 10 \times 20 = 1 \text{ J}$

b) $\Delta U_s = -\Delta U_g = -1 \text{ J}$

c) $\Delta U_s = -\frac{1}{2} k \Delta x^2$
 $+1 = +\frac{1}{2} k (6.4 \times 10^{-3})$

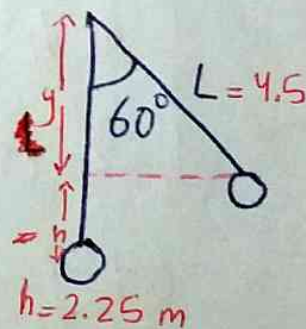
$k = 312.5 \frac{\text{N}}{\text{m}}$

20) $m = 2 \text{ Kg}$, $V_i = 8 \text{ m/s}$
 $L = 4.5 \text{ m}$

$\cos 60^\circ = \frac{1}{2}$

$\frac{y}{4.5} = \frac{1}{2}$

$y = 2.25 \text{ m}$



بعد تعويض الأرقام

$V_f = 4.3 \text{ m/s}$

a) $E_{mec} = E_{mec}$
 $K_i + U_i = K_f + U_f$
 $\frac{1}{2} m V_i^2 = \frac{1}{2} m V_f^2 + mgh$



(b)

$0 = K_f \leftarrow 0 = V_f$ تكون الزاوية "θ" أكبر ما يمكن عندما

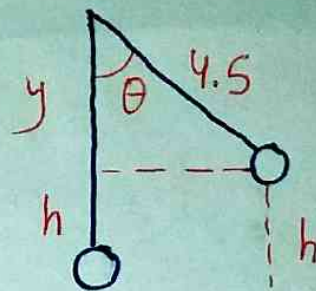
$$E_{mec\ i} = E_{mec\ f}$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m V_i^2 = mgh$$

$$32 = 10h$$

$$h = 3.2$$



$$\cos \theta = \frac{y}{4.5}$$

$$y = 4.5 - h$$

$$\cos \theta = \frac{4.5 - h}{4.5}$$

$$\cos \theta = \frac{4.5 - 3.2}{4.5}$$

$$\cos \theta = 0.288$$

$$\theta = 73.2^\circ$$

(c)

$$E_{mec\ i} = K_i + U_i$$

$$= \frac{1}{2} m V_i^2$$

$$= \frac{1}{2} \cdot 2 \cdot (8)^2$$

$$= 64 \text{ J}$$

23) $L = 1.2 \text{ m}$, $d = 0.75 \text{ m}$

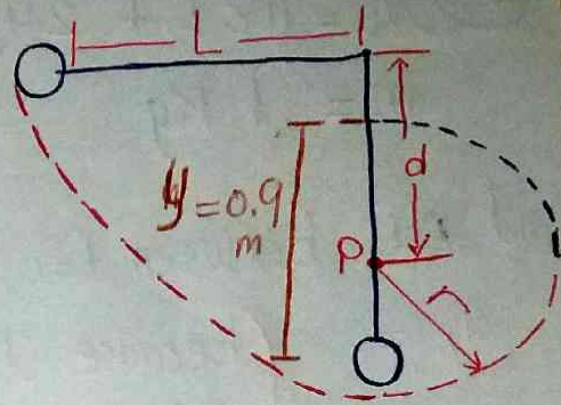
a) $E_{mec\ i} = E_{mec\ f}$

$U_i + K_i = U_f + K_f$

$mgh = \frac{1}{2} m V_f^2$

$12 = \frac{1}{2} V_f^2$

$V_f = 4.9 \text{ m/s}$



$L - d = 1.2 - 0.75$
 $= 0.45 = r$

$y = 2r = 0.9 \text{ m}$

b) $E_{mec\ i} = E_{mec\ f}$

$U_i + K_i = U_f + K_f$

$mgh = mgy + \frac{1}{2} m V_f^2$

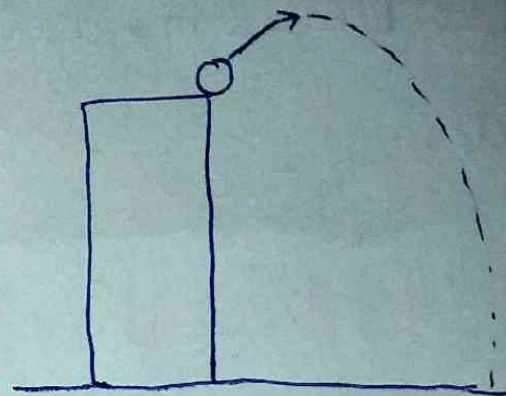
$12 = 9 + \frac{1}{2} V_f^2$

$V_f^2 = 6$

$V_f = 2.45 \text{ m/s}$

25) $V_i = (18\hat{i} + 24\hat{j}) \text{ m/s}$ at $t=0$

$m = 1 \text{ Kg}$



find ΔU between $t=0$ and $t=6$?

$a_x = 0$ (because its free fall and no acceleration on x-axis)

★ $V_{fx} = V_{ix} = 18\hat{i} \text{ m/s}$

★ $V_{fy} = V_{iy} - gt$

$V_{fy} = 24 - 60$
 $= -36\hat{j} \text{ m/s}$

$V_f = \sqrt{(18)^2 + (-36)^2} = 40.25 \text{ m/s}$

$V_i = \sqrt{(18)^2 + (24)^2} = 30 \text{ m/s}$

$\Delta U = -\Delta K$

$\Delta U = -\left(\frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2\right)$

$= \frac{1}{2} m V_i^2 - \frac{1}{2} m V_f^2$

$= \frac{1}{2} m (V_i^2 - V_f^2)$

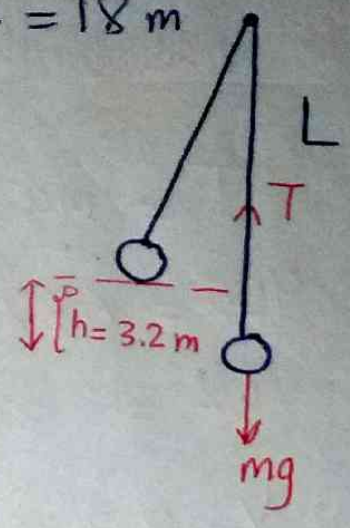
$= \frac{1}{2} (900 - 1620)$

$= -360 \text{ J}$

(27) $mg = 688 \text{ N}$, $h = 3.2 \text{ m}$, $L = 18 \text{ m}$

(a) $T - mg = \frac{mV^2}{r}$

$$T = mg + \frac{mV^2}{r} \quad \text{--- (1)}$$



$E_{mec} = E_{mec}$

~~U_i + K_i = U_f + K_f~~

$U_i + K_i = U_f + K_f$

$mgh = \frac{1}{2} m V^2$

$V^2 = 64$

$V = 8 \text{ m/s}$

$$T = 688 + \frac{68.8 \times 64}{18}$$

$$= 932.6 \text{ N}$$

then the vine doesn't break

(b) the greatest force = 932.6 N

~~(c)~~

(c) \Rightarrow

© The vine will break if $T = 950 \text{ N}$

$$T = mg + \frac{mV^2}{r}$$

$$950 = 688 + \frac{68.8 V^2}{18}$$

$$262 = 3.82 V^2$$

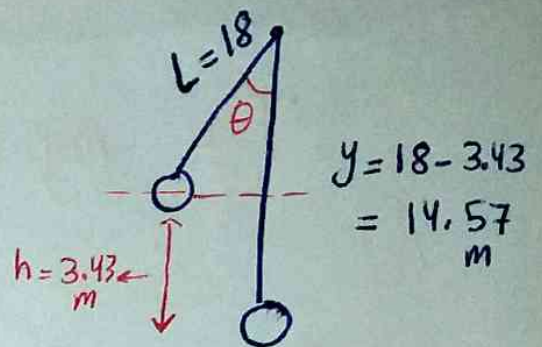
$$V = 8.3 \text{ m/s}$$

$$E_{mec} = E_{mec}$$

$$U_i + K_i = U_f + K_f$$

$$mgh = \frac{1}{2} mV^2$$

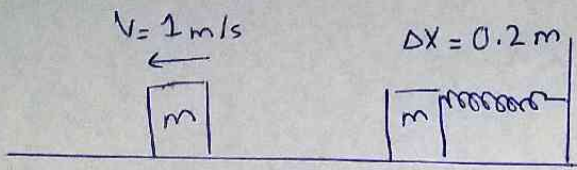
$$h = 3.43 \text{ m}$$



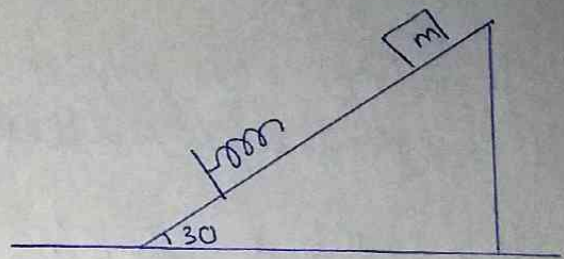
$$\cos \theta = \frac{14.57}{18}$$

$$\theta = 36^\circ$$

(29) $m = 4 \text{ kg}$



(1)



(2)

From figure (1) $\Rightarrow E_{mec_i} = E_{mec_f} \Rightarrow U_s + K_i \overset{\rightarrow}{=} K_f$
 $\frac{1}{2} K (\Delta x)^2 = \frac{1}{2} m v^2$
 $0.04 K = 4$
 $K = 100 \frac{\text{N}}{\text{m}}$

(a) from figure (2) :-

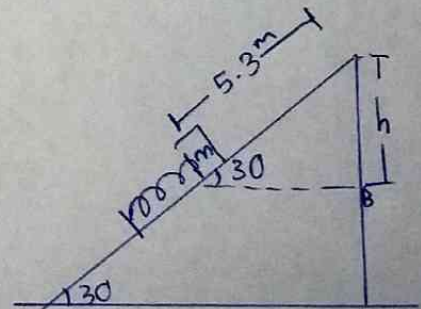
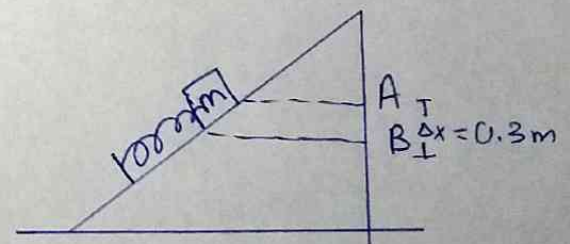
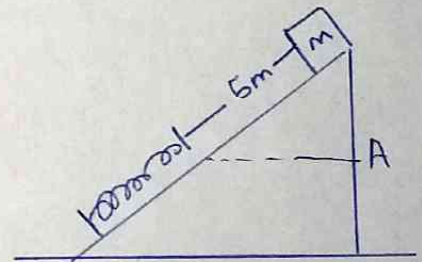
$E_{mec_i} = E_{mec_f}$

$U_i = U_s + M_s mg \cos \theta d$

$mgh = \frac{1}{2} K (\Delta x)^2 + M_s (40 \cos 30 \times 3.5)$

$106 = 4.5 + 183.5 M_s$

$M_s = 0.55$



PS $\Rightarrow \sin 30 = \frac{1}{2}$

$\frac{h}{5.3} = \frac{1}{2}$

$h = 2.65 \text{ m}$

$$(b) E_{mec\ i} = E_{mec\ f}$$

$$U_s + \cancel{U_i} + \cancel{K_i} = \cancel{U_s} + U_f + \cancel{K_f} + \Delta E_{th}$$

$$4.5 = mgh + \mu_s mg \cos 30 d$$

$$4.5 = mg h + 0.55 * 4 * 10 * 0.87 * 2h$$

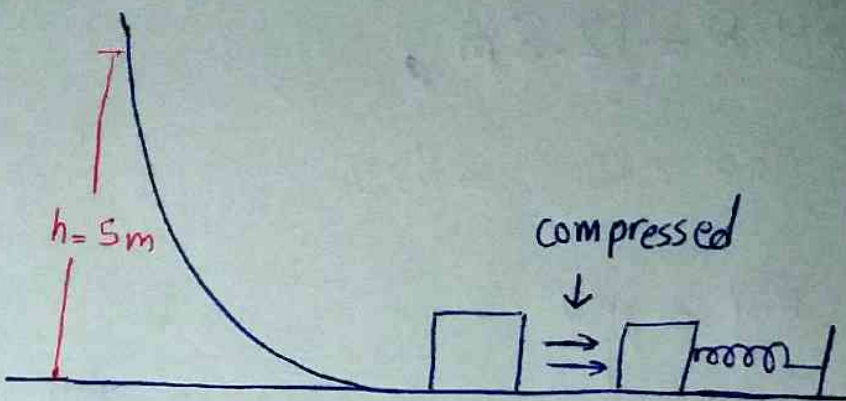
$$4.5 = 40h + 38.28 h$$

$$h = 0.057$$

$$d = 2h = 0.1149 \text{ m}$$

31 $\Delta x = 0.25 \text{ m}$
 $m = 1 \text{ kg}$

a)



$$E_{mec\ i} = E_{mec\ f}$$

$$U_i + K_i = U_f + K_f$$

$$mgh = \frac{1}{2} K \Delta x^2$$

$$\frac{50}{\Delta x^2} = \frac{1}{2} K$$

$$K = \frac{100}{(0.25)^2} = 16000 \frac{\text{N}}{\text{m}}$$

c) No change, because E_{mec} is conservation

b) $E_{mec\ i} = E_{mec\ f}$

$$U_i + K_i = U_f + K_f$$

$$50 = \frac{1}{2} m V_f^2$$

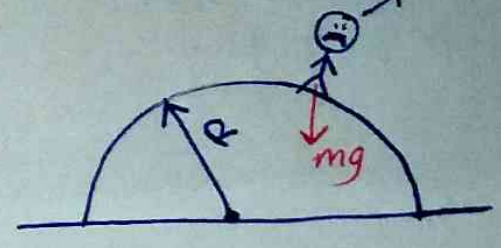
$$V_f^2 = 100$$

$$V = 10 \text{ m/s}$$

34) $R = 12.8 \text{ m}$

نفسان "أب عقل"

lose contact $\Rightarrow N = 0$



$$mg = \frac{mV^2}{r}$$

$$V^2 = 128 \text{ m}^2/\text{s}^2$$

~~Energy~~ $E_{mec} = E_{mec}$

$$U_i + K_i = U_f + K_f$$

$$mgh_1 = mgh_2 + \frac{1}{2}mV^2$$

$$128 = 10h_2 + 64$$

$$h_2 = 6.4 \text{ m}$$

38) $E_{mec} = U_B + K_B = 16 \text{ J}$

a) $x = 3.5$

$$\begin{cases} \rightarrow U = 9 \text{ J} \\ \rightarrow K = 16 - 9 = 7 \text{ J} \end{cases}$$

$$K = \frac{1}{2}mV^2$$

$$7 = 0.1V^2 \quad \Rightarrow$$

$$V^2 = 70$$

$$V = 8.36 \text{ m/s}$$

(b) $x = 6.5 \text{ m}$

$$\begin{array}{l} \rightarrow U = 0 \\ \rightarrow K = 16 \text{ J} \end{array}$$

$$K = \frac{1}{2} m v^2$$

$$16 = 0.1 v^2$$

$$v = \sqrt{160} = 12.6 \text{ m/s}$$

(c) slope₁ = slope₂

$$\frac{24 - 16}{8 - x_t} = \frac{16 - 0}{x_t - 7}$$

x_t when $\Delta K = 0$

$$U = E = 16 \text{ J}$$

$$\frac{8}{8 - x_t} = \frac{16}{x_t - 7}$$

$$x_t - 7 = 16 - 2x_t$$

$$3x_t = 23$$

$$x_t = 7.67$$

(d) slope₁ = slope₂

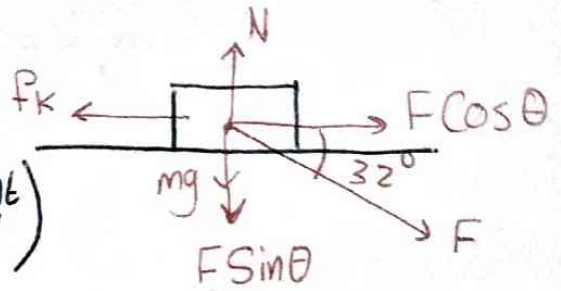
$$\frac{20 - 16}{1 - x_t} = \frac{16 - 9}{x_t - 3}$$

$$\frac{4}{1 - x_t} = \frac{7}{x_t - 3}$$

$$7 - 7x_t = 4x_t - 12$$

$$11x_t = 19 \rightarrow x_t = 1.73$$

42) $m = 23 \text{ Kg}$, $\mu_k = 0.2$
 $\theta = 32^\circ$, $d = 8.4 \text{ m}$



(a) $F \cos \theta - f_k = m \vec{a}$ (because constant speed)

$$F \cos \theta = f_k$$

$$F \cos \theta = \mu_k F \sin \theta + \mu_k mg \quad (\theta = 32^\circ) \quad (N = mg + F \sin \theta)$$

$$0.85 F = 0.11 F + 46$$

$$F = 62.2 \text{ N}$$

$$W = F d \cos \theta$$

$$= 62.2 \times 8.4 \times \cos(32)$$

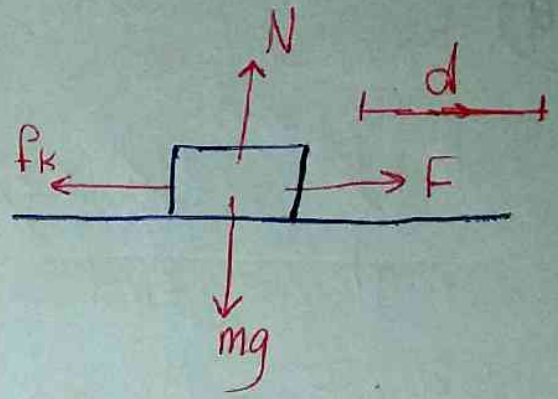
$$= 443 \text{ J}$$

(b) $\Delta E_{\text{thermal}} = f_k d$ " $f_k = \mu_k F \sin \theta + \mu_k mg$ "

$$= 52.8 \times 8.4$$

$$= 443.5 \text{ J}$$

44) $\mu_k = 0.6$, $F = 41 \text{ N}$
 $m = 4 \text{ Kg}$, $d = 2 \text{ m}$



a) $W = \vec{F} \cdot \vec{d}$
 $= Fd \cos \theta$
 $= 41 \times 2 \times \cos(0)$
 $= 82 \text{ J}$

b) $\Delta E_{\text{thermal}} = f_k d \cos(180)$
 $= -\mu_k mg d$
 $= -0.6 \times 4 \times 10 \times 2$
 $= -48 \text{ J}$

← طاقة مفقودة
 (لا تكوّن سالب عند العمل)

c) $W = \Delta K + \Delta E_{\text{thermal}}$
 $82 = K_f - K_i + 40$
 $K_f - K_i = 42$
 $\Delta K = 42 \text{ J}$

49

$$E_{mec\ i} = E_{mec\ f}$$

$$U_i + K_i = U_f + K_f + \Delta E_{thermal}$$

$$mgh = f_k d$$

$$520 \times 7 = \mu_k mg d$$

$$3640 = 156 d$$

$$d = 23.3 \text{ m}$$

★ P.S: $m = 50 + 2$
 $= 52 \text{ Kg}$
M.S

62

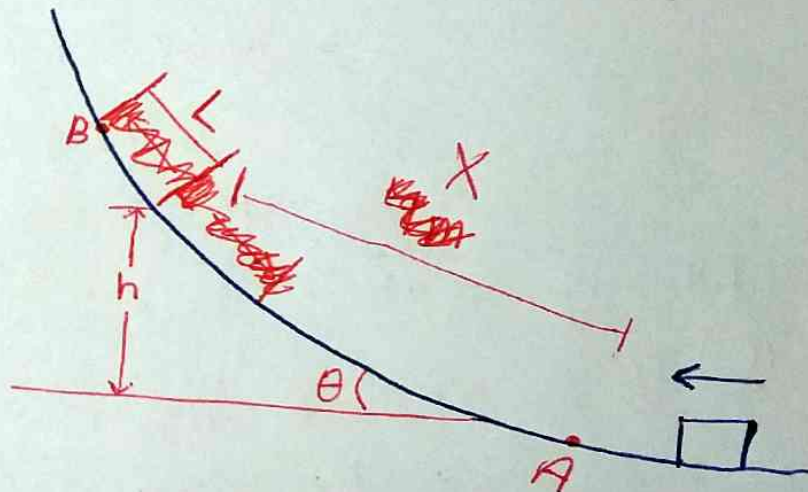
$$L = 0.65 \text{ m}$$

$$\theta = 30^\circ$$

$$h = 2 \text{ m}$$

$$\mu_k = 0.4$$

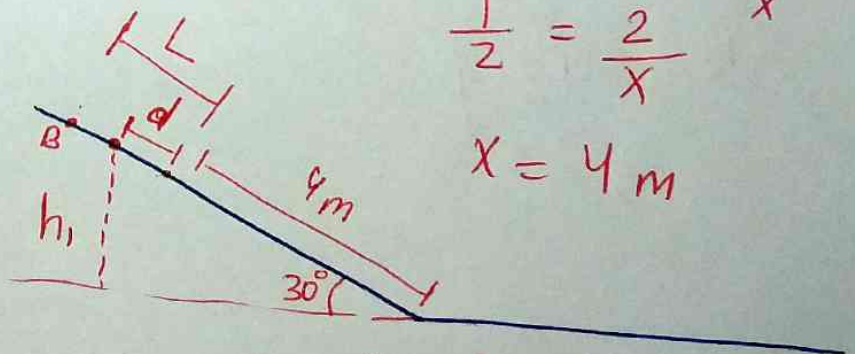
$$V_0 = 8 \text{ m/s}$$



$$\sin 30^\circ = \frac{h}{x}$$

$$\frac{1}{2} = \frac{2}{x}$$

$$x = 4 \text{ m}$$



$$\sin 30^\circ = \frac{h_1}{4+d}$$

$$\frac{1}{2} = \frac{h_1}{4+d}$$

$$h_1 = 2 + \frac{1}{2}d$$

a) $E_{mec_i} = E_{mec_f} \Rightarrow U_i + K_i = U_f + K_f + \Delta E_{ther}$

$\frac{1}{2} m v_i^2 = mgh_1 + \mu_k m g \cos 30^\circ d$

$32 = 20 + 5d + (0.4 \times 10 \times \frac{\sqrt{3}}{2}) d$

$12 = 5d + 2\sqrt{3}d$

$12 = 8.4d$

$d = 1.41 \text{ m}$

the particle will pass the point B



b)

$E_{mec_i} = E_{mec_f}$

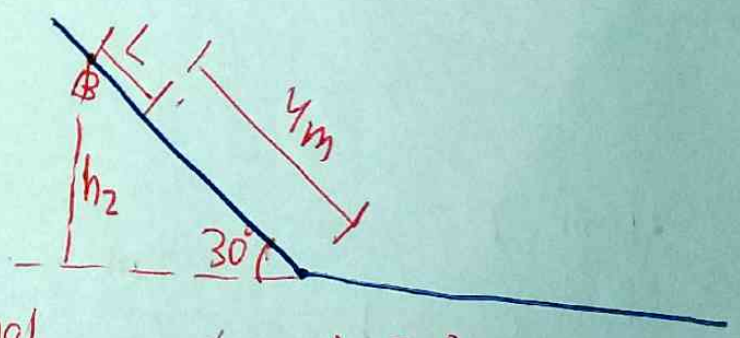
$U_i + K_i = U_f + K_f + \Delta E_{thermal}$

$\frac{1}{2} m v_i^2 = mgh_2 + \frac{1}{2} m v_f^2 + \mu_k m g \cos 30^\circ d$

$32 = 23.2 + \frac{1}{2} v_f^2 + 2.25$

$v_f^2 = 12.7$

$v_f = 3.5 \text{ m/s}$



$\sin 30^\circ = \frac{h_2}{4.65}$

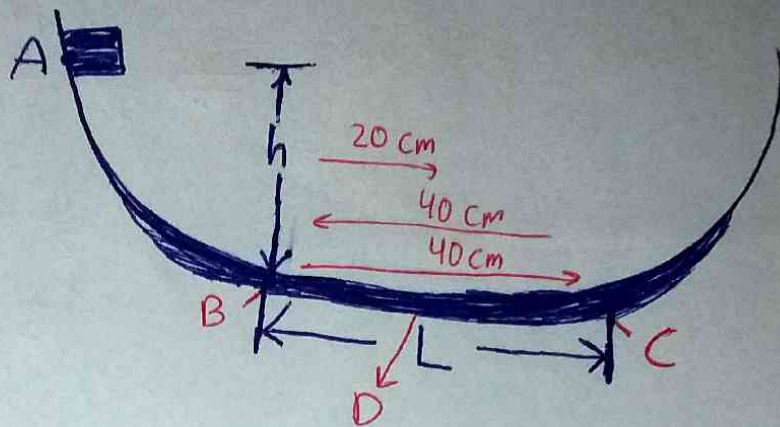
$\frac{1}{2} = \frac{h_2}{4.65}$

$h_2 = 2.32 \text{ m}$

65) $L = 0.4 \text{ m}$

$h = \frac{L}{2} = 0.2 \text{ m}$

$M_k = 0.2$



$E_{mec} = E_{mec}$

$U_i + K_i = U_f + K_f + \Delta E_{thermal}$

$mg \frac{L}{2} = M_k mg d$

$d = \frac{L}{2M_k} = \frac{0.4}{0.4} = 1$

$d = 1 \text{ m}$

و سوید
القنبر

- from B → C : 0.4 m

⇒ from C → B : 0.4 m

⇒ from B → D : 0.2 m

then the particle is located 0.2 m from the left

Chapter 9 "Center of Mass and linear momentum"

Lecture questions (1, 6, 13, 20, 32)

1

$$x_{com} = \frac{(0 \times 1) + (3 \times 1) + (2 \times 6.15)}{6} \quad m_c = (0.15, 0.87)$$

$$= \frac{4}{6} m$$

↓
 1 نفقار
 (تقووظ عورى)
 ↓
 عينا عورى
 $= \sqrt{0.15^2 + 0.87^2}$

$$y_{com} = \frac{1(0) + 3(0) + 2(0.87)}{6} = 0.29 m$$

$$\vec{r}_{com} = 0.167\hat{i} + 0.29\hat{j}$$

6

$$r_1 = (0 + \frac{L}{2}\hat{i} + \frac{L}{2}\hat{k}) \quad r_2 = (\frac{L}{2}\hat{i} + 0 + \frac{L}{2}\hat{k})$$

$$r_3 = (\frac{L}{2}\hat{i} + \frac{L}{2}\hat{j} + \frac{L}{2}\hat{k}) \quad r_4 = (\frac{L}{2}\hat{i} + L\hat{j} + \frac{L}{2}\hat{k})$$

$$r_5 = (\frac{L}{2}\hat{i} + \frac{L}{2}\hat{j} + 0)$$

$$L = 50 cm = 0.5 m$$

$$\vec{r}_{com} = \frac{1}{5m'} (m_1\vec{r}_1 + \dots + m_5\vec{r}_5)$$

$$\vec{r}_{com} = \frac{L}{5} (\frac{5}{2}L\hat{i} + \frac{5}{2}L\hat{j} + 2L\hat{k})$$

$$\vec{r}_{com} = \frac{L}{2}\hat{i} + \frac{L}{2}\hat{j} + \frac{2L}{5}\hat{k}$$

13

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

من قوانين الكتوفعات

	x	y
v_0	$20 \cos 60$	$20 \sin 60$

v	$20 \cos 60$	0
-----	--------------	---

a	0	-9.8
-----	---	------

t	1.767	1.767
-----	-------	-------

Δx	17.67	15.3m
------------	-------	-------

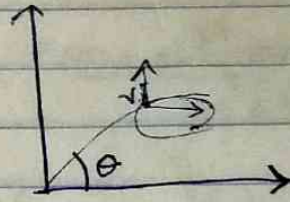
$$\Delta y = 20 \sin 60 (1.767) + \frac{1}{2}(-9.8)(1.767^2)$$

$$\Delta y = 15.3m$$

20

$$P_0 = 6 \quad P_1 = 4$$

$$\theta = ?$$



~~$$\Delta P = \int$$~~

$$\cos \theta = \frac{P_1}{P_0} = \frac{4}{6}$$

~~ثابت~~

ثابت السرعة الأفقية
ثابتة

$$\theta = \cos^{-1}(0.67) \\ = 47.93^\circ$$

32

a) $P?$ at $t=4s$

$\Delta P = \int$ Area under the curve of $(F \text{ vs } t)$

$$P - P_0 = \frac{1}{2}(2)(10) + 2(10) = 30 \text{ kg} \cdot \text{m/s}$$

b) $P = ?$ at $t = 7s$

$$P - 0 = 30 + \frac{1}{2}(2)(10) + \frac{1}{2}(-5)(1)$$

$$P = 37.5 \text{ kg} \cdot \text{m/s} \quad (+X)$$

c) $v = ?$ at $t = 9s$

$$P - P_0 = 37.5 + 1(-5) + \frac{1}{2} \times 1 \times (-5) = m \cdot v$$

$$30 = \overset{2.5}{m} \cdot v \rightarrow v = 12 \text{ m/s} \quad \underline{\underline{+X}}$$

الكتلة الكوبية لعمود السيارة ←

discussion problems (4, 5, 16, 25, 35, 52, 59)

$$4) X_{com} = \frac{X_1 m_1 + X_2 m_2 + X_3 m_3}{m_1 + m_2 + m_3} = \frac{0 m_1 + \frac{24}{2} \cdot 42 + 24 \cdot 14}{42 + 14 + 14} = 12 \text{ cm}$$

$$y_{com} = \frac{-\frac{24 \cdot 14}{2} + 0 + -\frac{24 \cdot 14}{2}}{42 + 14 + 14} = -4.8 \text{ cm}$$

$$5) X_{com} = \frac{2L \cdot 4 + -L \cdot 6 + -L \cdot 8 + L \cdot 4}{22} = -0.09L = -0.45 \text{ cm}$$

$$y_{com} = \frac{2 \cdot 5L \cdot 4 + 1.5L \cdot 6 + -2 \cdot 8L + -3 \cdot 4}{22}$$

$$\delta = \frac{M}{A} \leftarrow \text{الضغط السطحي ثابت}$$

$$m_1 = \delta \cdot 4L^2 \hat{i} \hat{j} \hat{k}$$

مركز الكتلة في المنتصف *

16)

$$X_{com} = \frac{m_g x_g + m_c x_c + m_b x_b}{\sum m}$$

$$X_{com} = \frac{m_g x_0 + 30 \times 1.5 + 80 \times 3}{m_g + 80 + 30}$$

$$X_{com} = \frac{2.85}{m_g + 110}$$

$$\hat{X}_{com} = \frac{m_g \hat{x}_g + m_c \hat{x}_c + m_b \hat{x}_b}{\sum m}$$

$$X_{com} = \frac{m_g \cdot (3 - 0.45) + 30 \times (1.5 - 0.45) - 80 \cdot (0.45)}{m_g + 30 + 80}$$

$$= \frac{2.85}{m_g + 110} \quad \dots \quad \frac{m_g \cdot 0.55}{m_g + 110}$$

25) $m = 5 \text{ g}$ $v_0 = 100 \text{ m/s}$ $d = 6.00 \text{ cm}$

a) $v^2 = v_0^2 + 2a \Delta x$
 $(100)^2 = 2a \times 6 \times 10^{-2} \Rightarrow a = \frac{-10^4}{0.12} \text{ m/s}^2$

$v = v_0 + at \Rightarrow 0 = 100 - \frac{10^4}{0.12} t \Rightarrow t = 1.2 \text{ ms}$

b) $\Delta p = \bar{F} = m(v_f - v_i)$
 $= 5 \times 10^{-3} (0 - 100) = 0.5 \text{ kg} \cdot \text{m/s}$

c) $F = \frac{\Delta p}{\Delta t} = \frac{-5}{1.2 \times 10^{-3}} = 417 \text{ N}$

35) $m = 58 \text{ g}$ $v_0 = 34 \text{ m/s}$

$J = m(v_f - v_i) = 2mv_i$

$J = 2 \times 0.058 (\text{kg}) \times 34 = 3.944 \text{ N} \cdot \text{s}$

$J = \text{Area} \times \overset{g}{\uparrow} \underset{\rightarrow}{h_s} = 10^{-3} \text{ s}$

$3.944 = f_{\text{max}} (1 \times 10^{-3}) + f_{\text{max}} (2 \times 10^{-3}) + f_{\text{max}} (1 \times 10^{-3})$

$f_{\text{max}} = 986 \text{ N}$

52) $(m_b)(v_i)_b = (m_b)(v_f)_b$

$v = \frac{0.010 \times 1000 - 0.010 \times 400}{5}$

$v = 12 \text{ m/s}$

$\frac{1}{2} (m_{\text{block}}) v^2 = (m_{\text{block}}) gh$

$\frac{1}{2} \times 5 \times (12)^2 = 5 \times 9.8 \times h \rightarrow \underline{\underline{h = 7.3 \text{ cm}}}$

59) $W = \frac{1}{2} k x^2$

$$x = \sqrt{\frac{2W}{k}} = \sqrt{\frac{2(k \cdot E_f - k \cdot E_i)}{k}}$$

1)
$$v_f = \sqrt{\frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}}$$

$$\vec{P}_i = \vec{P}_f \quad (m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f)$$

2)
$$v_f = \frac{2(\text{kg}) \times 10 + 5 \times 3}{2 + 5} = 5 \text{ m/s}$$

$x = 25 \text{ cm} \leftarrow$ 1 is up for

Quiz questions (2, 9, 18, 29, 37, 69)

2)

a)
$$x_{\text{com}} = \frac{(2 \cdot 0) + (4 \cdot 2) + (8 \cdot 1)}{2 + 4 + 8} = 1.07 \text{ m}$$

b)

$$y_{\text{com}} = \frac{2 \cdot 0 + 4 \cdot 1 + 8 \cdot 2}{14} = 1.33 \text{ m}$$

c) if mass 3 increased, then centre of mass shifts towards the particle

9)

$$T - mg = ma$$

1) $T = a + g$

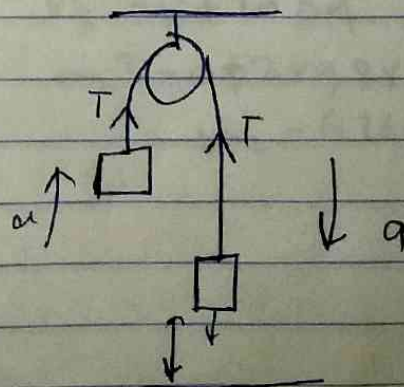
$$m_A \times g - T = m_A \times a$$

$$2g - T = 2 \times a$$

2) $T = 2g - 2a$

$$a = \frac{g}{3}$$

2) 1) do



a)
$$a_{com} = \frac{-g/3 \times 2 + g/3 \times 1}{2+1} = \frac{-g}{6} = -1.1 \text{ m/s}^2$$

b) displacement $= vt + \frac{1}{2}at^2$
 $v = 0 \quad t = 2 \text{ s} \quad a = -1.1$

$$= \frac{1}{2} \times -1.1 \times 2^2 = -2.2 \text{ m}$$

c) displacement when $h = 10 \text{ cm}$

$$10 = \frac{1}{2} \times (3.2) \times t^2 \rightarrow g/3$$

$$t = 2.475$$

$$v = v_0 + at \quad a = 1.1 \text{ m/s}^2 \quad t = 2.475$$

$$v_0 = 0$$

$$v = 1.1 \times 2.47 = 2.7 \text{ m/s}$$

18) $m = 0.7 \text{ kg} \quad v_0 = 6 \text{ m/s} \quad v_c = 3.5 \text{ m/s}$

$$|\Delta P| = |-3.5(0.7) - 6(0.7)| = 6.65 \text{ kg} \cdot \text{m/s}$$

29) $m = 1 \text{ kg} \quad d = 2 \text{ m}$

impulse $= J = \Delta P$

$$= 1(v_2 - 0)$$

$$J = 6.26 \text{ kg} \cdot \text{m/s}$$

$$v_2^2 = v_1^2 + 2ad$$

$$v_2^2 = 0 + 2 \times 9.8 \times 2$$

$$v_2 = 6.26 \text{ m/s}$$

37) $\vec{F} = 700 e^{-2t} \hat{i}$ $t=0s \rightarrow t=2s$

a) $J = ? = \int_{t_i}^{t_f} f(t) dt$
 $= \int_0^2 700 e^{-2t} dt$
 $= 100 \frac{e^{-2t}}{-2} \Big|_0^2$
 $= -50 (e^{-4} - 1)$
 $= 49.1 \hat{i} \text{ N.s}$

b) $F_{avg} \rightarrow J = F_{avg} \times \Delta t$
 $49.1 \hat{i} = F_{avg} \times (2-0)$
 $\rightarrow F_{avg} = 24.5 \hat{i} \text{ N}$

64) $mgh = \frac{1}{2} mv^2$
 $v = \sqrt{2gh} \rightarrow h = 0.7 \text{ m}$
 $\rightarrow v_1 = 3.7 \text{ m/s}$

a) $v_1' = \frac{m_1 - m_2}{m_1 + m_2} \times v_1$
 $v_1' = \frac{0.6 - 2.8}{0.6 + 2.8} \times 3.7 = -2.4 \text{ m/s}$

b) $v_2' = \frac{2m_1}{m_1 + m_2} \times v_1 \rightarrow v_2' = \frac{2 \times 0.6}{0.6 + 2.8} \times 3.7$
 $v_2' = 1.31 \text{ m/s}$

Good luck

Chapter ten

Q1.) $R = 1\text{ m}$
 $r = \frac{1}{4} = 0.25\text{ m}$
 $\omega_{0,R} = 100\text{ rev/min}$
 $\omega_{0,r} = ??$
 $d = ??$

a) $V_{\text{Rear}} = V_{\text{Front}} \Rightarrow R \omega_{0,R} = r \omega_{0,r}$
 $\Rightarrow (1)(100) = (\frac{1}{4})(\omega_{0,r})$
 $\Rightarrow \omega_{0,r} = 400\text{ rev/min}$

b) You can find the distance in 2 ways:

① $V = \frac{d}{t} \Rightarrow R \omega_{0,R} = \frac{d}{t}$
 $\Rightarrow (1)(10.47)(600) = d$
 $d = 6283.18\text{ m}$

$t = 10\text{ min} \Rightarrow 600\text{ sec}$
 $\omega_{0,R} = 100\text{ rev/min}$
 $\Rightarrow 10.47\text{ rad/s}$
 $\omega_{0,r} = 400\text{ rev/min}$
 $\Rightarrow 41.88\text{ rad/s}$

② $V = \frac{d}{t} \Rightarrow r \omega_{0,r} = \frac{d}{t}$
 $\Rightarrow (\frac{1}{4})(41.88)(600) = d \Rightarrow d = 6283.18\text{ m}$

Q5) $t = 0$

~~$V_{0,y} = 49 \text{ m/s}$~~

$V_{0,y} = 0$

~~$V_{f,y} = 49 \text{ m/s}$~~

$\theta = 40 \text{ rev/s} \Rightarrow 80\pi$

$\omega_{avg} = ?$

$V_{f,y} = V_{0,y} + g t \Rightarrow 49 = 0 + 9.8 t$
 $\Rightarrow t = 5$

$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \Rightarrow \frac{80\pi}{5} \Rightarrow 16\pi$

Q9)

$\Delta t = 5 \text{ s}$

$m = 2 \text{ kg}$

$r = 2 \text{ m}$

$\omega_0 = 0$

$\omega_f = 4 \text{ rev/s} \Rightarrow 8\pi$

$\alpha_{avg} = ??$

$I = ??$

$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} \Rightarrow \frac{8\pi}{5} \Rightarrow 5 \text{ rad/s}^2$

$I = m R^2 \Rightarrow 2(4) \Rightarrow 8 \text{ kg} \cdot \text{m}^2$

Q12)

$\omega_0 = 1200 \text{ rev/min}$, $\Delta t = 12 \text{ s}$ | $\alpha = ??$

$\omega_f = 3200 \text{ rev/min} \Rightarrow \Delta t = 0.2 \text{ min}$ | $\theta = ??$

~~$\omega_f = \omega_0 + \frac{1}{2} \alpha t^2$~~ $\omega_f = \omega_0 + \alpha t$

~~$\Rightarrow 3200 = 1200(0.2) + \frac{1}{2} \alpha (0.2)^2$~~ $\Rightarrow \alpha =$

$\Rightarrow 3200 = 1200 + \alpha(0.2) \Rightarrow \alpha = 10,000 \text{ rev/min}^2$

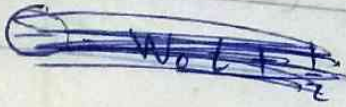
$\theta = \frac{1}{2} (\omega_f + \omega_0) t \Rightarrow \theta = \frac{1}{2} (335.1 + 125.7) (12)$

Note: convert ω_f , ω_0 to rad/s .

$\Rightarrow \theta = 2764.8 \text{ rad}$

Q13) $\Theta = 40 \text{ revs} \Rightarrow 80\pi$
 $\omega_0 = 1.5 \text{ rad/s}$
 $\omega_f = 0$

- | $t = ??$
- | $\alpha = ??$
- | time to complete
- | first 20 revs.



$$\omega_f^2 = \omega_0^2 + 2\alpha\Theta \Rightarrow 0 = (1.5)^2 + 2(\alpha)(80\pi)$$

$$\Rightarrow \alpha = -4.5 \times 10^{-3} \text{ rad/s}^2$$

$$\omega_f = \omega_0 + \alpha t \Rightarrow 0 = 1.5 + (-4.5 \times 10^{-3})(t)$$

$$\Rightarrow t = 335.1 \text{ s.}$$

c) time to complete first 20 revs? 40π

$$\Theta = \omega_0 t + \frac{1}{2} \alpha t^2 \Rightarrow 40\pi = (1.5)t + \frac{1}{2}(-4.5 \times 10^{-3})t^2$$

$$\Rightarrow -2.25 \times 10^{-3} t^2 + 1.5t - 40\pi = 0$$

$$t = 98.25 \text{ s}$$

or

$$t = 568.4 \text{ s} \times \Rightarrow \text{because (time to stop} = 335 \text{ s)}$$

Q16) $\alpha = 1.2 \text{ rad/s}^2$
 $\omega_0 = 0$

$4\pi = 0 + \frac{1}{2}(1.2)(t)^2$
 $t = 4.65$

$8\pi = 0 + \frac{1}{2}(1.2)(t)^2$
 $t = 6.4 \text{ s}$

4

time to rotate twice ($\theta = 2 \text{ revs}$).
 $\theta = 4\pi$
 time to rotate 4 times ($\theta = 4 \text{ revs}$).
 $\theta = 8\pi$

Q32) $V_0 = \omega_0 = 0$
 $R = 32 \text{ m}$
 $a_T = 0.6 \text{ m/s}^2$
 $t = 15 \text{ s}$

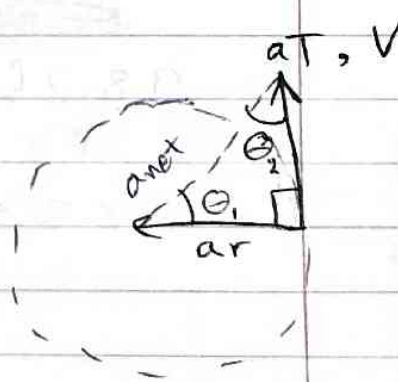
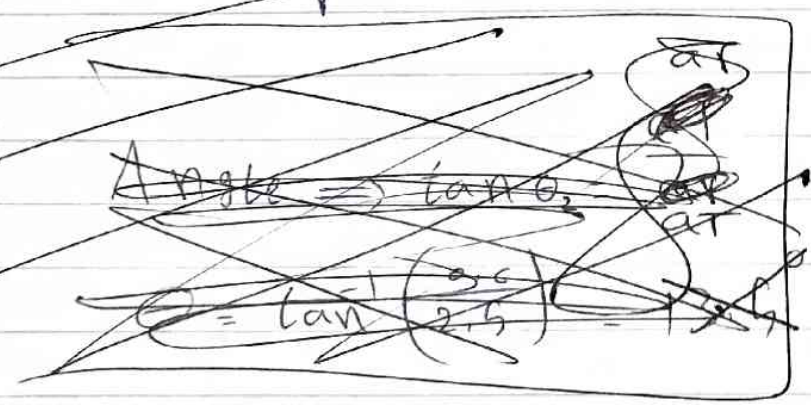
$a_{\text{net}} = ??$
 Angle between a_{net} and Velocity?

~~$V_f = \omega_f R$~~

$V_f = V_0 + (0.6)(15) \Rightarrow V_f = 9 \text{ m/s}$

$a_r = \frac{V^2}{R} \Rightarrow \frac{9}{32} \Rightarrow a_r = \text{~~0.28125 m/s}^2~~ 2.5 \text{ m/s}^2$

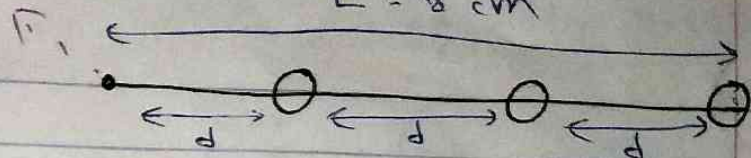
$a_{\text{net}} = \sqrt{(a_r)^2 + (a_T)^2} = 2.6 \text{ m/s}^2$



Angle - $\tan \theta_2 = \frac{a_r}{a_T}$

$\theta_2 = \tan^{-1} \left(\frac{2.5}{0.6} \right) \Rightarrow 76^\circ$

Q38)



$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2$$

$$= md^2(1 + 4 + 9) \Rightarrow 14md^2$$

a) Percentage if we remove inner most one?

$$I_{\text{new}} = m(2d)^2 + m(3d)^2$$

$$= md^2(4 + 9) \Rightarrow 13md^2$$

$$\text{Percentage} = \frac{14md^2 - 13md^2}{14md^2} \times 100 = 7.1\%$$

b) Percentage if ~~inner~~^{outermost} is removed?

$$I_{\text{new}} = md^2(1 + 4) \Rightarrow 5md^2$$

$$\text{Percentage} = \frac{14md^2 - 5md^2}{14md^2} \times 100 = 64\%$$

Q45) $F_1 = 4.2$ $r_1 = 1.3$ $\theta_1 = 75^\circ$ $\tau_{\text{net}} = ??$
 $F_2 = 4.9$ $r_2 = 2.15$ $\theta_2 = 60^\circ$

$$\tau_{\text{net}} = \tau_1 - \tau_2$$

$$\tau_1 = F_1 r_1 \sin \theta_1 \Rightarrow 4.2(1.3) \sin 75$$

$$\tau_2 = F_2 r_2 \sin \theta_2 \Rightarrow 4.9(2.15) \sin 60$$

$$\tau_{\text{net}} = -3.85 \text{ N}\cdot\text{m} \Rightarrow \text{Clock wise.}$$

49)

$$\tau = F_{\text{net}} R$$

$$F_{\text{net}} = mg - T \Rightarrow T = mg - ma$$

$$I = \frac{1}{2} M R^2$$

$$\alpha = \frac{a}{R}$$

$$\Rightarrow I \alpha = F_{\text{net}} R \Rightarrow \left(\frac{1}{2} M R^2\right) \left(\frac{a}{R}\right) = (mg - ma)(R)$$

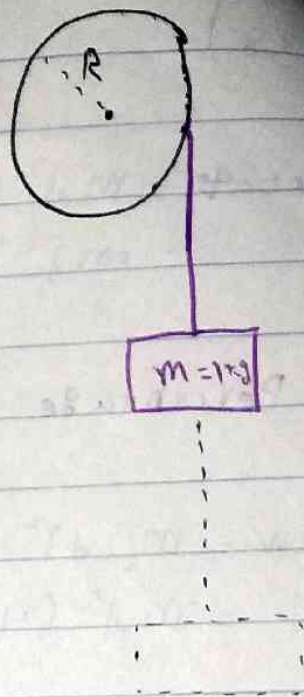
$$a = \frac{mg}{m + \frac{1}{2}M} \Rightarrow a = \frac{9.8}{1 + 2.5} \Rightarrow 2.8 \text{ m/s}^2$$

$$\alpha = \frac{a}{R} \Rightarrow \alpha = 9.3 \text{ rad/s}^2$$

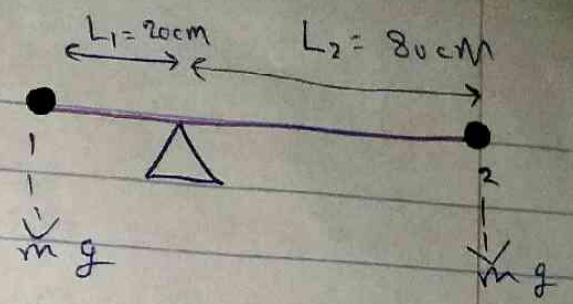
$$\theta = 0 + \frac{1}{2} \alpha t^2 \Rightarrow \frac{1}{2} (9.3) (25) \Rightarrow 116.25 \text{ rads} \Rightarrow 18.5 \text{ revs.}$$

50) $\tau = 42 \text{ N}\cdot\text{m}$
 $\alpha = 25 \text{ rad/s}^2$

$$\tau = I \alpha \Rightarrow I = \frac{42}{25} \Rightarrow 1.68 \text{ kg}\cdot\text{m}^2$$



Q 56) $m_1 = m_2 = m$
 $L_1 = 0.2 \text{ m}$
 $L_2 = 0.8 \text{ m}$



Find the acceleration of both objects?

$$\tau_{\text{net}} = \tau_1 - \tau_2$$

$$\tau_1 = m g L_1$$

$$\tau_2 = m g L_2$$

$$\tau_{\text{net}} = \tau_1 - \tau_2$$

$$I_1 = m L_1^2$$

$$I_2 = m L_2^2$$

$$(m L_1^2 + m L_2^2) \alpha = m g (L_1 - L_2)$$

$$\alpha = 8.65 \text{ rad/s}^2$$

$$a_1 = L_1 \alpha \Rightarrow a_1 = 1.73 \text{ m/s}^2$$

$$a_2 = L_2 \alpha \Rightarrow a_2 = 6.92 \text{ m/s}^2$$

59) $L = 30 \text{ m}$ $V = ??$
 $m = 100 \text{ kg}$

$$m g \frac{L}{2} = \frac{1}{2} \left(\frac{1}{3} m R^2 \right) \frac{V^2}{R^2}$$

$$g L = \frac{1}{3} V^2 \Rightarrow V = \sqrt{9.8 (30)(3)}$$

$$\Rightarrow V = 29.7 \text{ m/s}$$



$$I = \frac{1}{3} m R^2$$

$$w = \frac{V}{R}$$

~~V before hitting~~

